IV. *On the Power of penetrating into Space by Telescopes; with a comparative Determination of the Extent of that Power in natural Vision, and in Telescopes of various Sizes and Constructions; illustrated by select Observations. By* William Herschel, *LL. D. F. R. S.*

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It will not be difficult to shew that the power of penetrating into space by telescopes is very different from magnifying power, and that, in the construction of instruments, these two powers ought to be considered separately.

In order to conduct our present inquiry properly, it will be necessary to examine the nature of luminous bodies, and to enter into the method of vision at a distance. Therefore, to prevent the inaccuracy that would unavoidably arise from the use of terms in their common acceptation, I shall have recourse to algebraic symbols, and to such definitions as may be necessary to fix a precise meaning to some expressions which are often used in conversation, without much regard to accuracy.

By luminous bodies I mean, in the following pages, to denote such as throw out light, whatever may be the cause of it: even those that are opaque, when they are in a situation to reflect light, should be understood to be included; as objects of vision they must throw out light, and become intitled to be called luminous. However, those that shine by their own light may be called self-luminous, when there is an occasion to distinguish them.

The question will arise, whether luminous bodies scatter light in all directions equally; but, till we are more intimately acquainted with the powers which emit and reflect light, we shall probably remain ignorant on this head. I should remark, that what I mean to say, relates only to the physical points into which we may conceive the surfaces of luminous bodies to be divided; for, when we take any given luminous body in its whole construction, such as the sun or the moon, the question will assume another form, as will appear hereafter.

That light, flame, and luminous gases are penetrable to the rays of light, we know from experience;* it follows therefore, that every part of the sun's disk cannot appear equally luminous to an observer in a given situation, on account of the unequal depth of its luminous atmosphere in different places.† This regards only bodies that are self-luminous. But the greatest inequalities in the brightness of luminous bodies in general, will undoubtedly be owing to their natural texture;

* In order to put this to a proof, I placed four candles behind a screen, at $\frac{3}{4}$ of an inch distance from each other, so that their flames might range exactly in a line. The first of the candles was placed at the same distance from the screen, and just opposite a narrow slit, $\frac{2}{3}$ of an inch long, and $\frac{1}{4}$ broad. On the other side of the screen I fixed up a book, at such a distance from the slit that, when the first of the candles was lighted, the letters might not be sufficiently illuminated to become legible. Then, lighting successively the second, third, and fourth candles, I found the letters gradually more illuminated, so that at last I could read them with great facility; and, by the arrangement of the screen and candles, the light of the second, third, and fourth, could not reach the book, without penetrating the flames of those that were placed before them.

† See the Paper on the Nature and Construction of the Sun. Phil. Trans. for 1795, page 46.

which may be extremely various, with regard to their power of throwing out light more or less copiously.

Brightness, I ascribe to bodies that throw out light; and those that throw out most are the brightest.

It will now be necessary to establish certain expressions for brightness in different circumstances.

In the first place, let us suppose a luminous surface throwing out light, and let the whole quantity of light thrown out by it be called L.

Now, since every part of this surface throws out light, let us suppose it divided into a number of luminous physical points, denoted by N.

If the copiousness of the emission of light from every physical point of the luminous surface were equal, it might in general be denoted by c; but, as that is most probably never the case, I make C stand for the mean copiousness of light thrown out from all the physical points of a luminous object. This may be found in the following manner. Let c express the copiousness of emitting light, of any number of physical points that agree in this respect; and let the number of these points be n. Let the copiousness of emission of another number of points be c', and their number n'. And if, in the same manner, other degrees of copiousness be called c^2, c^3, &c. and their numbers be denoted by n^2, n^3, &c. then will the sum of every set of points, multiplied by their respective copiousness of emitting light, give us the quantity of light thrown out by the whole luminous body. That is, $L = cn + c'n' + c^2n^2$, &c.; and the mean copiousness of emitting light, of each physical point, will be expressed by $\frac{cn + c'n' + c^2n^2, \&c.}{N} = C$.

It is evident that the mean power, or copiousness of throwing out light, of every physical point in the luminous surface, multiplied by the number of points, must give us the whole power of throwing out light, of the luminous body. That is $CN = L$.

I ought now to answer an objection that may be made to this theory. Light, as has been stated, is transparent; and, since the light of a point behind the surface of a flame will pass through the surface, ought we not to take in its depth, as well as its superficial dimensions? In answer to this, I recur to what has been said with regard to the different powers of throwing out light, of the points of a luminous surface. For, as light must be finally emitted through the surface, it is but referring all light arising from the emission of points behind the surface, to the surface itself, and the account of emitted light will be equally true. And this will also explain why it has been stated as probable, that different parts of the same luminous surface may throw out different quantities of light.

Since, therefore, the quantity of light thrown out by any luminous body is truly represented by CN, and that an object is bright in consequence of light thrown out, we may say that brightness is truly defined by CN. If however, there should at any time be occasion for distinction, the brightness arising from the great value of C, may be called the intrinsic brightness; and that arising from the great value of N, the aggregate brightness; but the absolute brightness, in all cases, will still be defined by CN.

Hitherto we have only considered luminous objects, and their condition with regard to throwing out light. We proceed now to find an expression for their appearance at any assigned distance; and here it will be proper to leave out of the account, every part of CN which is not applied for the purpose of vision. L representing the whole quantity of light thrown out by CN, we shall denote that part of it which is used in vision, either by the eye or by the telescope, l. This will render the conclusions that may be drawn hereafter more unexceptionable; for, the quantity of light l being scattered over a small space in proportion to L, it may reasonably be looked upon as more uniform in its texture; and no scruples about its inequality will take place. The equation of light, in this present sense, therefore, is $CN = l$.

Now, since we know that the density of light decreases in the ratio of the squares of the distances of the luminous objects, the expression for its quantity at the distance of the observer D, will be $\frac{l}{D^2}$.

In natural vision, the quantity l undergoes a considerable change, by the opening and contracting of the pupil of the eye. If we call the aperture of the iris a, we find that in different persons it differs considerably. Its changes are not easily to be ascertained; but we shall not be much out in stating its variations to be chiefly between 1 and 2 tenths of an inch. Perhaps this may be supposed under-rated; for the powers of vision, in a room completely darkened, will exert themselves in a very extraordinary manner. In some experiments on light, made at Bath, in the year 1780, I have often remarked, that after staying some time in a room fitted up for these experiments, where on entering I could not perceive any one object, I was no longer at a loss, in half an hour's time, to find every thing I wanted. It is however probable that the opening of the iris is not the only cause of seeing better after remaining long in the dark; but that the tranquillity of the retina, which is not disturbed by foreign objects of vision, may render it fit to receive impressions such as otherwise would have been too faint to be perceived. This seems to be supported by telescopic vision; for it has often happened to me, in a fine winter's evening, when, at midnight, and in the absence of the moon, I have taken sweeps of the heavens, of four, five, or six hours duration, that the sensibility of the eye, in consequence of the exclusion of light from surrounding objects, by means of a black hood which I wear upon these occasions, has been very great; and it is evident, that the opening of the iris would have been of no service in these cases, on account of the diameter of the optic pencil, which, in the 20 feet telescope, at the time of sweeping, was no more than ,12 inch. The effect of this increased sensibility was such, that if a star of the 3d magnitude came towards the field of view, I found it necessary to withdraw the eye before its entrance, in order not to injure the delicacy of vision acquired by long continuance in the dark. The transit of large stars, unless where none of the 6th or 7th magnitude could be had, have generally been declined in my sweeps, even with the 20 feet telescope. And I remember, that after a considerable sweep with the 40 feet instrument, the appearance of Sirius announced itself, at a great distance, like the dawn of the morning, and came on by degrees, increasing in brightness, till this brilliant star at last entered the field of view of the telescope, with all the splendour of the rising sun, and forced me to take the eye from that beautiful sight. Such striking effects are a sufficient proof of the great sensibility of the eye, acquired by keeping it from the light.

On taking notice, in the beginning of sweeps, of the time that passed, I found that the eye, coming from the light, required near 20′, before it could be sufficiently reposed to admit a view of very delicate objects in the telescope; and that the observation of a transit of a star of the 2d or 3d magnitude, would disorder the eye again, so as to require nearly the same time for the re-establishment of its tranquillity.

The difficulty of ascertaining the greatest opening of the eye, arises from the impossibility of measuring it at the time of its extreme dilatation, which can only happen when every thing is completely dark; but, if the variation of a is not easily to be ascertained, we have, on the other hand, no difficulty to determine the quantity of light admitted through a telescope, which must depend upon the diameter of the object-glass, or mirror; for, its aperture A may at all times be had by measurement.

It follows, therefore, that the expression $\frac{a^2 l}{D^2}$ will always be accurate for the quantity of light admitted by the eye; and that $\frac{A^2 l}{D^2}$ will be sufficiently so for the telescope. For it must be remembered, that the aperture of the eye is also concerned in viewing with telescopes; and that, consequently, whenever the pencil of light transmitted to the eye by optical instruments exceeds the aperture of the pupil, much light must be lost. In that case, the expression $A^2 l$ will fail; and therefore, in general, if m be the magnifying power, $\frac{A}{m}$ ought not to exceed a.

As I have defined the brightness of an object to the eye of an observer at a distance, to be expressed by $\frac{a^2 l}{D^2}$, it will be necessary to answer some objections that may be made to this theory. Optical writers have proved, that an object is equally bright at all distances. It may, therefore, be maintained against me, that since a wall illuminated by the sun will appear equally bright, at whatsoever distance the observer be placed that views it; the sun also, at the distance of Saturn, or still farther from us, must be as bright as it is in its present situation. Nay, it may be urged, that in a telescope, the different distance of stars can be of no account with regard to their brightness, and that we must consequently be able to see stars which are many thousands of times farther than Sirius from us; in short, that a star must be infinitely distant not to be seen any longer.

Now, objections such as these, which seem to be the immediate consequence of what has been demonstrated by mathematicians, and which yet apparently contradict what I assert in this paper, deserve to be thoroughly answered.

It may be remembered, that I have distinguished brightness into three different sorts.* Two of these, which have been discriminated by *intrinsic* and *absolute* brightness, are, in common language, left without distinction. In order to shew that they are so, I might bring a variety of examples from common conversation; but, taking this for granted, it may be shewn that all the objections I have brought against my theory have their foundation in this ambiguity.

The demonstrations of opticians, with regard to what I call *intrinsic* brightness, will not oppose what I affirm of *absolute* brightness; and I shall have nothing farther to do than to shew that what mathematicians have said, must be understood to refer entirely to the intrinsic brightness, or illumination of the picture of objects on the retina of the eye: from which it will clearly follow, that their doctrine and mine are perfectly reconcileable; and that they can be at variance only when the ambiguity of the word brightness is overlooked, and objections,

* See page 52.

such as I have made, are raised, where the word brightness is used as *absolute*, when we should have kept it to the only meaning it can bear in the mathematicians' theorem.

The first objection I have mentioned is, that the sun, to an observer on Saturn, must be as bright as it is here on earth. Now by this cannot be meant, that an inhabitant standing on the planet Saturn, and looking at the sun, should *absolutely* receive as much light from it as one on earth receives when he sees it; for this would be contrary to the well known decrease of light at various distances. The objection, therefore, can only go to assert, that the picture of the sun, on the retina of the Saturnian observer, is as *intensely* illuminated as that on the retina of the terrestrial astronomer. To this I perfectly agree. But let those who would go farther, and say that therefore the sun is *absolutely* as bright to the one as to the other, remember that the sun on Saturn appears to be a hundred times less than on the earth; and that consequently, though it may there be *intrinsically* as bright, it must here be *absolutely** an hundred times brighter.

The next objection I have to consider, relates to the fixed stars. What has been shewn in the preceding paragraph, with regard to the sun, is so intirely applicable to the stars, that it will be very easy to place this point also in its proper light. As I have assented to the demonstration of opticians with regard to the brightness of the sun, when seen at the distance of Saturn, provided the meaning of this word be kept to the *intrinsic* illumination of the picture on the retina of an observer, I can have no hesitation to allow that the same will hold good with a star placed at any assignable distance. But I must repeat, that

* See the definition of *absolute* brightness, page 52.

the light we can receive from stars is truly expressed by $\frac{a^2 l}{D^2}$; and that therefore their absolute brightness must vary in the inverse ratio of the squares of their distances. Hence I am authorised to conclude, and observation abundantly confirms it, that stars cannot be seen by the naked eye, when they are more than seven or eight times farther from us than Sirius; and that they become, comparatively speaking, very soon invisible with our best instruments. It will be shewn hereafter, that the visibility of stars depends on the penetrating power of telescopes, which, I must repeat, falls indeed very short of shewing stars that are many thousands of times farther from us than Sirius; much less can we ever hope to see stars that are all but infinitely distant.

If now it be admitted that the expressions we have laid down are such as agree with well known facts, we may proceed to vision at a distance; and first with respect to the naked eye.

Here the power of penetrating into space, is not only confined by nature, but is moreover occasionally limited by the failure in brightness of luminous objects. Let us see whether astronomical observations, assisted by mathematical reasoning, can give us some idea of the general extent of natural vision. Among the reflecting luminous objects, our penetrating powers are sufficiently ascertained. From the moon we may step to Venus, to Mercury, to Mars, to Jupiter, to Saturn, and last of all to the Georgian planet. An object seen by reflected light at a greater distance than this, it has never been allowed us to perceive; and it is indeed much to be admired, that we should see borrowed illumination to the amazing distance of more than 18 hundred millions of miles; especially when that light, in coming from the sun to the planet, has to pass through an equal space, before it can be reflected, whereby it must be so enfeebled as to be above 368 times less intense on that planet than it is with us, and when probably not more than one-third part of that light can be thrown back from its disk.*

The range of natural vision with self-luminous objects, is incomparably more extended, but less accurately to be ascertained. From our brightest luminary, the sun, we pass immediately to very distant objects; for, Sirius, Arcturus, and the rest of the stars of the first magnitude, are probably those that come next; and what their distance may be, it is well known, can only be calculated imperfectly from the doctrine of parallaxes, which places the nearest of them at least 412530 times farther from us than the sun.

In order to take a second step forwards, we must enter into some preliminary considerations, which cannot but be attended with considerable uncertainty. The general supposition, that stars, at least those which seem to be promiscuously scattered, are probably one with another of a certain magnitude, being admitted, it has already been shewn in a former Paper,† that after a certain number of stars of the first magnitude have been arranged about the sun, a farther distant set will come in for the second place. The situation of these may be taken to be, one with another, at about double the distance of the former from us.

* According to Mr. Bouguer, the surface of the moon absorbs about two-thirds of the light it receives from the sun. See *Traité d'Optique*, page 122.

† Phil. Trans. for the year 1796, page 166, 167, 168.

By directing our view to them, and thus penetrating one step farther into space, these stars of the second magnitude furnish us with an experiment that shews what phænomena will take place, when we receive the illumination of two very remote objects, equally bright in themselves, whereof one is at double the distance of the other. The expression for the brightness of such objects, at all distances, and with any aperture of the iris, according to our foregoing notation, will be $\frac{a^2 l}{D^2}$; and a method of reducing this to an experimental investigation will be as follows.

Let us admit that α Cygni, β Tauri, and others, are stars of the second magnitude, such as are here to be considered. We know, that in looking at them and the former, the aperture of the iris will probably undergo no change; since the difference in brightness, between Sirius, Arcturus, α Cygni, and β Tauri, does not seem to affect the eye so as to require any alteration in the dimensions of the iris; *a*, therefore becomes a given quantity, and may be left out. Admitting also, that the latter of these stars are probably at double the distance of the former, we have D^2 in one case four times that of the other; and the two expressions for the brightness of the stars, will be l for those of the first magnitude, and $\frac{1}{4}l$ for those of the second.

The quantities being thus prepared, what I mean to suggest by an experiment is, that since sensations, by their nature, will not admit of being halved or quartered, we come thus to know by inspection what phænomenon will be produced by the fourth part of the light of a star of the first magnitude. In this sense, I think we must take it for granted, that a certain idea of brightness, attached to the stars which are generally denominated to be of the second magnitude, may be added to our experimental knowledge; for, by this means, we are informed what we are to understand by the expressions $\frac{a^2 l}{\odot^2}$, $\frac{a^2 l}{\overline{\text{Sirius}}|^2}$, $\frac{a^2 l}{\overline{\beta\ \text{Tauri}}|^2}$.* We cannot wonder at the immense difference between the brightness of the sun and that of Sirius; since the two first expressions, when properly resolved, give us a ratio of brightness of more than 170 thousand millions to one; whereas the two latter, as has been shewn, give only a ratio of four to one.

What has been said will carry us, with very little addition, to the end of our unassisted power of vision to penetrate into space. We can have no other guide to lead us a third step than the same beforementioned hypothesis; in consequence of which, however, it must be acknowledged to be sufficiently probable, that the stars of the third magnitude may be placed about three times as far from us as those of the first. It has been seen, by my remarks on the comparative brightness of the stars, that I place no reliance on the classification of them into magnitudes;† but, in the present instance, where the question is not to ascertain the precise brightness of any one star, it is quite sufficient to know that the number of the stars of the first three different magnitudes, or different brightnesses, answers, in a general way, sufficiently well to a supposed equally distant arrangement of a first, second, and third set of stars about the sun. Our third step forwards into space, may therefore very properly be said to fall on the pole-star, on γ Cygni, ϵ Bootis, and all those of the same order.

* The names of the objects ⊙, Sirius, β Tauri, are here used to express their distance from us.

† Phil. Trans. for the year 1796, page 168, 169.

As the difference, between these and the stars of the preceding order, is much less striking than that between the stars of the first and second magnitude, we also find that the expressions $\frac{a^2 l}{\beta \text{ Tauri}\}^2}$, and $\frac{a^2 l}{\text{Polaris}\}^2}$, are not in the high ratio of 4 to 1, but only as 9 to 4, or $2\frac{1}{4}$ to 1.

Without tracing the brightness of the stars through any farther steps, I shall only remark, that the diminution of the ratios of brightness of the stars of the 4th, 5th, 6th, and 7th magnitude, seems to answer to their mathematical expressions, as well as, from the first steps we have taken, can possibly be imagined. The calculated ratio, for instance, of the brightness of a star of the 6th magnitude, to that of one of the 7th, is but little more than $1\frac{1}{3}$ to 1; but still we find by experience, that the eye can very conveniently perceive it. At the same time, the faintness of the stars of the 7th magnitude, which require the finest nights, and the best common eyes to be perceived, gives us little room to believe that we can penetrate much farther into space, with objects of no greater brightness than stars.

But, since it may be justly observed, that in the foregoing estimation of the proportional distance of the stars, a considerable uncertainty must remain, we ought to make a proper allowance for it; and, in order to see to what extent this should go, we must make use of the experimental sensations of the ratios of brightness we have now acquired, in going step by step forward: for, numerical ratios of brightness, and sensations of them, as has been noticed before, are very different things. And since, from the foregoing considerations, it may be concluded, that as far as the 6th, 7th, or 8th magnitude, there ought to be a visible general difference between stars of one order and that of the next following, I think, from the faintness of the stars of the 7th magnitude, we are authorized to conclude, that no star, eight, nine, or at most ten times as far from us as Sirius, can possibly be perceived by the natural eye.

The boundaries of vision, however, are not confined to single stars. Where the light of these falls short, the united lustre of sidereal systems will still be perceived. In clear nights, for instance, we may see a whitish patch in the sword-handle of Perseus,* which contains small stars of various sizes, as may be ascertained by a telescope of a moderate power of penetrating into space. We easily see the united lustre of them, though the light of no one of the single stars could have affected the unassisted eye.

Considerably beyond the distance of the former must be the cluster discovered by Mr. Messier, in 1764; north following H Geminorum. It contains stars much smaller than those of the former cluster; and a telescope should have a considerable penetrating power, to ascertain their brightness properly, such as my common 10-feet reflector. The night should be clear, in order to see it well with the naked eye, and it will then appear in the shape of a small nebula.

Still farther from us must be the nebula between η and ζ Herculis, discovered by Dr. Halley, in 1714. The stars of it are so small that it has been called a Nebula;† and has been regarded as such, till my instruments of high penetrating

* See the catalogue of a second thousand of new nebulæ and clusters of stars, VI. 33, 34. Phil. Trans. Vol. LXXIX. page 251.

† In the *Connoissance des Temps* for 1783, No. 13, it is described as a nebula without stars.

powers were applied to it. It requires a very clear night, and the absence of the moon, to see it with the natural eye.

Perhaps, among the farthest objects that can make an impression on the eye, when not assisted by telescopes, may be reckoned the nebula in the girdle of Andromeda, discovered by SIMON MARIUS, in 1612. It is however not difficult to perceive it, in a clear night, on account of its great extent.

From the powers of penetrating into space by natural vision, we proceed now to that of telescopes.

It has been shewn, that brightness, or light, is to the naked eye truly represented by $\frac{a^2 l}{D^2}$; in a telescope, therefore, the light admitted will be expressed by $\frac{A^2 l}{D^2}$. Hence it would follow, that the artificial power of penetrating into space should be to the natural one as A to a. But this proportion must be corrected by the practical deficiency in light reflected by mirrors, or transmitted through glasses; and it will in a great measure depend on the circumstances of the workmanship, materials, and construction of the telescope, how much loss of light there will be sustained.

In order to come to some determination on this subject, I made many experiments with plain mirrors, polished like my large ones, and of the same composition of metal. The method I pursued was that proposed by Mr. BOUGUER, in his *Traité d'Optique*, page 16, fig. 3.; but I brought the mirror, during the trial, as close to the line connecting the two objects as possible, in order to render the reflected rays nearly perpendicular. The result was, that out of 100 thousand incident rays, 67262 were returned; and therefore, if a double reflection takes place, only 45242 will be returned.

Before this light can reach the eye, it will suffer some loss in passing through the eye glass; and the amount of this I ascertained, by taking a highly polished plain glass, of nearly the usual thickness of optical glasses of small focal lengths. Then, by the method of the same author, page 21, fig. 5. I found, that out of 100 thousand incident rays, 94825 were transmitted through the glass. Hence, if two lenses be used, 89918; and, with three lenses, 85265 rays will be transmitted to the eye.

Then, by compounding, we shall have, in a telescope of my construction with one reflection, 63796 rays, out of 100 thousand, come to the eye. In the NEWTONIAN form, with a single eye lens, 42901; and, with a double eye glass 40681 will remain for vision.

There must always remain a considerable uncertainty in the quantities here assigned; as a newly polished mirror, or one in high preservation, will give more light than another that has not those advantages. The quality of metal also will make some difference; but, if it should appear by experiments, that the metals or glasses in use will yield more or less light than here assigned, it is to be understood that the corrections must be made accordingly.

We proceed now to find a proper expression for the power of penetrating into space, that we may be enabled to compare its effects, in different telescopes, with that of the natural eye.

Since then the brightness of luminous objects is inversely as the squares of the distances, it follows, that the penetrating power must be as the square roots of the light received by the eye.

In natural vision, therefore, this power is truly expressed by $\sqrt{a^2 l}$; and, since we have now also obtained a proper correction x, we must apply it to the incident light with telescopes.

In the NEWTONIAN and other constructions where two specula are used, there will also be some loss of light on account of the interposition of the small speculum; therefore, putting b for its diameter, we have $\overline{A^2 - b^2}$ for the real incident light This being corrected as above, will give the general expression $\sqrt{xl \times \overline{A^2 - b^2}}$ for the same power in telescopes. But here we are to take notice, that in refractors, and in telescopes with one reflection, b will be $= o$, and therefore is to be left out.

Then, if we put natural light $l = 1$, and divide by a, we have the general form $\frac{\sqrt{x \cdot \overline{A^2 - b^2}}}{a}$ for the penetrating power of all sorts of telescopes, compared to that of the natural eye as a standard, according to any supposed aperture of the iris, and proportion of light returned by reflection, or transmitted by refraction.

In the following investigation we shall suppose $a = 2$ tenths of an inch, as being perhaps nearly the general opening of the iris, in star-light nights, when the eye has been some moderate time in the dark. The value of the corrections for loss of light will stand as has been given before.

We may now proceed to determine the powers of the instruments that have been used in my astronomical observations; but, as this subject will be best explained by a report of the observations themselves, I shall select a series of them for that purpose, and relate them in the order which will be most illustrating.

First, with regard to the eye, it is certain that its power, like all our other faculties, is limited by nature, and is regulated by the permanent brightness of objects: as has been shewn already, when its extent with reflected light was compared to its exertion on self-luminous objects. It is further limited on borrowed light, by the occasional state of illumination; for, when that becomes defective at any time, the power of the eye will then be contracted into a narrower compass; an instance of which is the following.

In the year 1776, when I had erected a telescope of 20 feet focal length, of the NEWTONIAN construction, one of its effects by trial was, that when towards evening, on account of darkness, the natural eye could not penetrate far into space, the telescope possessed that power sufficiently to shew, by the dial of a distant church steeple, what o'clock it was, notwithstanding the naked eye could no longer see the steeple itself. Here I only speak of the penetrating power; for, though it might require magnifying power to see the figures on the dial, it could require none to see the steeple. Now the aperture of the telescope being 12 inches, and the construction of the NEWTONIAN form, its penetrating power, when calculated according to the given formula, will be $\frac{\sqrt{,429 \times 120^2 - 15^2}}{2} = 38,99$. *A*, *b*, and *a*, being all expressed in tenths of an inch.*

* I have given the figures, in all the following equations of the calculated penetrating powers, in order to shew the constructions of my instruments to those who may wish to be acquainted with them.

From the result of this computation it appears, that the circumstance of seeing so well, in the dusk of the evening, may be easily accounted for, by a power of this telescope to penetrate 39 times farther into space than the natural eye could reach, with objects so faintly illuminated.

This observation completely refutes an objection to telescopic vision, that may be drawn from what has also been demonstrated by optical writers; namely, that no telescope can shew an object brighter than it is to the naked eye. For, in order to reconcile this optical theory with experience, I have only to say, that the objection is intirely founded on the same ambiguity of the word brightness that has before been detected. It is perfectly true, that the *intrinsic* illumination of the picture on the retina, which is made by a telescope, cannot exceed that of natural vision; but the *absolute* brightness of the magnified picture by which telescopic vision is performed, must exceed that of the picture in natural vision, in the same ratio in which the area of the magnified picture exceeds that of the natural one; supposing the *intrinsic* brightness of both pictures to be the same. In our present instance, the steeple and clock-dial were rendered visible by the increased absolute brightness of the object, which in natural vision was 15 hundred times inferior to what it was in the telescope. And this establishes beyond a doubt, that telescopic vision is performed by the absolute brightness of objects; for, in the present case, I find by computation, that the *intrinsic* brightness, so far from being equal in the telescope to that of natural vision, was inferior to it in the ratio of three to seven.

The distinction between magnifying power, and a power of penetrating into space, could not but be felt long ago, though its theory has not been inquired into. This undoubtedly gave rise to the invention of those very useful short telescopes called night-glasses. When the darkness of the evening curtails the natural penetrating power, they come in very seasonably, to the relief of mariners that are on the look out for objects which it is their interest to discover. Night-glasses, such as they are now generally made, will have a power of penetrating six or seven times farther into space than the natural eye. For, by the construction of the double eye-glass, these telescopes will magnify 7 or 8 times; and the object glass being $2\frac{1}{2}$ inches in diameter, the breadth of the optic pencil will be $3\frac{1}{8}$ or $3\frac{4}{7}$ tenths of an inch. As this cannot enter the eye, on a supposition of an opening of the iris of 2 tenths, we are obliged to increase the value of a, in order to make the telescope have its proper effect. Now, whether nature will admit of such an enlargement becomes an object of experiment; but, at all events, a cannot be assumed less than $\frac{A}{m}$. Then, if x be taken as has been determined for three refractions, we shall have $\frac{\sqrt{.853 \times 25^2}}{a} = 6{,}46$ or $7{,}39$.

Soon after the discovery of the Georgian planet, a very celebrated observer of the heavens, who has added considerably to our number of telescopic comets and nebulæ, expressed his wish, in a letter to me, to know by what method I had been led to suspect this object not to be a star, like others of the same appearance. I have no doubt but that the instrument through which this astronomer generally looked out for comets, had a penetrating power much more than sufficient to shew the new planet, since even the natural eye will reach it. But here we have an instance of the great difference in the effect of the two sorts of powers of telescopes; for, on account of the smallness of the planet, a different sort of power, namely, that of magnifying, was required: and, about the time of its discovery, I had been remarkably attentive to an improvement of this power, as I happened to be then much in want of it for my very close double stars.*

On examining the nebulæ which had been discovered by many celebrated authors, and comparing my observations with the account of them in the *Connoissance des Temps* for 1783, I found that most of those which I could not resolve into stars with instruments of a small penetrating power, were easily resolved with telescopes of a higher power of this sort; and, that the effect was not owing to the magnifying power I used upon these occasions, will fully appear from the observations; for, when the closeness of the stars was such as to require a considerable degree of magnifying as well as penetrating power, it always appeared plainly, that the instrument which had the highest penetrating power resolved them best, provided it had as much of the other power as was required for the purpose.

Sept. 20, 1783, I viewed the nebula between FLAMSTEED's 99th and 105th Piscium, discovered by Mr. MECHAIN, in 1780. "It is not visible in the finder of my 7-feet telescope; but "that of my 20-feet shews it."

Oct. 28, 1784, I viewed the same object with the 7-feet telescope.

"It is extremely faint. With a magnifying power of 120, it "seems to be a collection of very small stars: I see many of "them."

* Magnifying powers of 460, 625, 932, 1159, 1504, 2010, 2398, 3168, 4294, 5489, 6450, 6652, were used upon ε Bootis, γ Leonis, α Lyræ, &c. See Cat. of double stars, Phil. Trans. Vol. LXXII. page 115, and 147; and Vol. LXXV. page 48.

At the time of these observations, my 7-feet telescope had only a common finder, with an aperture of the object glass of about $\frac{3}{4}$ of an inch in diameter, and a single eye-lens; therefore its penetrating power was $\frac{\sqrt{,899 \times \overline{7,5}|^2}}{2} = 3,56$. The finder of the 20-feet instrument, being achromatic, had an object glass 1,17 inch in diameter; its penetrating power, therefore, was $\frac{\sqrt{,85 \times \overline{11,7}|^2}}{2} = 4,50$.

Now, that one of them shewed the nebula and not the other, can only be ascribed to space-penetrating power, as both instruments were equal in magnifying power, and that so low as not to require an achromatic object glass to render the image sufficiently distinct.

The 7-feet reflector evidently reached the stars of the nebula; but its penetrating and magnifying powers are very considerable, as will be shewn presently.

July 30, 1783, I viewed the nebula south preceding FLAMSTEED's 24 Aquarii, discovered by Mr. MARALDI, in 1746.

" In the small *sweeper*,* this nebula appears like a telescopic " comet."

Oct. 27, 1794. The same nebula with a 7-feet reflector.

* The small sweeper is a NEWTONIAN reflector, of 2 feet focal length; and, with an aperture of 4,2 inches, has only a magnifying power of 24, and a field of view 2° 12'. Its distinctness is so perfect, that it will shew letters at a moderate distance, with a magnifying power of 2000; and its movements are so convenient, that the eye remains at rest while the instrument makes a sweep from the horizon to the zenith.

A large one of the same construction has an aperture of 9,2 inches, with a focal length of 5 feet 3 inches. It is also charged low enough for the eye to take in the whole optic pencil; and its penetrating power, with a double eye glass, is $\frac{\sqrt{,41 \times \overline{92^2 - 21^2}}}{2} = 28,57$.

"I can see that it is a cluster of stars, many of them being "visible."

If we compare the penetrating power of the two instruments, we find that we have in the first $\frac{\sqrt{.41 \times 42^2 - 12^2}}{2} = 12,84$; and in the latter $\frac{\sqrt{.41 \times 63^2 - 12^2}}{2} = 20,25$. However, the magnifying power was partly concerned in this instance; for, in the *sweeper* it was not sufficient to separate the stars properly.

March 4, 1783. With a 7-feet reflector, I viewed the nebula near the 5th Serpentis, discovered by Mr. MESSIER, in 1764.

"It has several stars in it; they are however so small that I "can but just perceive some, and suspect others."

May 31, 1783. The same nebula with a 10-feet reflector; penetrating power $\frac{\sqrt{.41 \times 89^2 - 16^2}}{2} = 28,67$.

"With a magnifying power of 250, it is all resolved into "stars: they are very close, and the appearance is beautiful. "With 600, perfectly resolved. There is a considerable star not "far from the middle; another not far from one side, but out "of the cluster; another pretty bright one; and a great number "of small ones."

Here we have a case where the penetrating power of 20 fell short, when 29 resolved the nebula completely. This object requires also great magnifying power to shew the stars of it well; but that power had before been tried, in the 7-feet, as far as 460, without success, and could only give an indication of its being composed of stars; whereas the lower magnifying power of 250, with a greater penetrating power, in the 10-feet instrument, resolved the whole nebula into stars.

May 3, 1783. I viewed the nebula between η and ρ Ophiuchi, discovered by Mr. MESSIER, in 1764.

"With a 10-feet reflector, and a magnifying power of 250, "I see several stars in it, and make no doubt a higher power, "and more light, will resolve it all into stars. This seems to be "a good nebula for the purpose of establishing the connection "between nebulæ and clusters of stars in general."

June 18, 1784. The same nebula viewed with a large NEWTONIAN 20-feet reflector; penetrating power $\frac{\sqrt{.43 \times 188^2 - 21^2}}{2} = 61,18$; and a magnifying power of 157.

"A very large and very bright cluster of excessively compressed stars. The stars are but just visible, and are of unequal magnitudes: the large stars are red; and the cluster is "a miniature of that near FLAMSTEED's 42d Comæ Berenices. "RA $17^h\ 6'\ 32''$; PD $108°\ 18'$."

Here, a penetrating power of 29, with a magnifying power of 250, would barely shew a few stars; when, in the other instrument, a power 61 of the first sort, and only 157 of the latter, shewed them completely well.

July 4, 1783. I viewed the nebula between FLAMSTEED's 25 and 26 Sagittarii, discovered by ABRAHAM IHLE, in 1665.

"With a small 20-feet NEWTONIAN telescope, power 200, "it is all resolved into stars, that are very small and close. "There must be some hundreds of them. With 350, I see the "stars very plainly; but the nebula is too low in this latitude "for such a power."

July 12, 1784. I viewed the same nebula with a large 20-feet NEWTONIAN reflector; power 157.

"A most beautiful extensive cluster of stars, of various magnitudes, very compressed in the middle, and about 8′ in "diameter, besides the scattered ones, which do more than fill "the extent of the field of view:* the large stars are red; the "small ones are pale red. RA 18h 23′ 39″; PD 114° 7′."

The penetrating power of the first instrument was 39, that of the latter 61; but, from the observations, it is plain how much superior the effect of the latter was to that of the former, notwithstanding the magnifying power was so much in favour of the instrument with the small penetrating power.

July 30, 1783. With a small 20-feet NEWTONIAN reflector, I viewed the nebula in the hand of Serpentarius, discovered by Mr. MESSIER, in 1764.

"With a power of 200, I see it consists of stars. They are "better visible with 300. With 600, they are too obscure to be "distinguished, though the appearance of stars is still preserved. "This seems to be one of the most difficult objects to be "resolved. With me, there is not a doubt remaining; but "another person, in order to form a judgment, ought previously "to go through all the several gradations of nebulæ which I "have resolved into stars."

May 25, 1791. I viewed the same nebula with a 20-feet reflector of my construction, having a penetrating power of

$$\frac{\sqrt{,64 \times 188|^2}}{2} = 75{,}08.$$

"With a magnifying power of 157, it appears extremely "bright, round, and easily resolvable. With 300, I can see the "stars. It resembles the cluster of stars taken at 16h 43′ 40″,†

* This field, by the passage of an equatorial star, was 15′ 3″.

† The object referred to is No. 10. of the *Connoissance des Temps* for 1783, called "*Nebuleuse sans etoiles.*" My description of it is, "A very beautiful, and extremely "compressed, cluster of stars: the most compressed part about 3 or 4′ in diameter. "RA 16h 46′ 2″; PD 93° 46′."

" which probably would put on the same appearance as this, " if it were at a distance half as far again as it is. RA 17^h 26′ " 19″; PD 93° 10′."

Here we may compare two observations; one taken with the penetrating power of 39, the other with 75; and, although the former instrument had far the advantage in magnifying power, the latter certainly gave a more complete view of the object.

The 20-feet reflector having been changed from the NEWTONIAN form to my present one, I had a very striking instance of the great advantage of the increased penetrating power, in the discovery of the Georgian satellites. The improvement, by laying aside the small mirror, was from 61 to 75; and, whereas the former was not sufficient to reach these faint objects, the latter shewed them perfectly well.

March 14, 1798. I viewed the Georgian planet with a new 25-feet reflector. Its penetrating power is $\frac{\sqrt{,64 \times \overline{240}^{2}}}{2} = 95,85$; and, having just before also viewed it with my 20-feet instrument, I found, that with an equal magnifying power of 300, the 25-feet telescope had considerably the advantage of the former.

Feb. 24, 1786. I viewed the nebula near FLAMSTEED's 5th Serpentis, which has been mentioned before, with my 20-feet reflector; magnifying power 157.

" The most beautiful extremely compressed cluster of small " stars; the greatest part of them gathered together into one " brilliant nucleus, evidently consisting of stars, surrounded " with many detached gathering stars of the same size and " colour. RA 15^h 7′ 12″; PD 87° 8′."

May 27, 1791. I viewed the same object with my 40-feet telescope; penetrating power $\frac{\sqrt{,64 \times 480|^2}}{2} = 191,69$; magnifying power 370.

"A beautiful cluster of stars. I counted about 200 of them. "The middle of it is so compressed that it is impossible to distinguish the stars."

Here it appears, that the superior penetrating power of the 40-feet telescope enabled me even to count the stars of this nebula. It is also to be noticed, that the object did not strike me as uncommonly beautiful; because, with much more than double the penetrating, and also more than double the magnifying power, the stars could not appear so compressed and small as in the 20-feet instrument: this, very naturally, must give it more the resemblance of a coarser cluster of stars, such as I had been in the habit of seeing frequently.

The 40-feet telescope was originally intended to have been of the NEWTONIAN construction; but, in the year 1787, when I was experimentally assured of the vast importance of a power to penetrate into space, I laid aside the work of the small mirror, which was then in hand, and completed the instrument in its present form.

"Oct. 10, 1791. I saw the 4th satellite and the ring of "Saturn, in the 40-feet speculum, without an eye glass."

The magnifying power on that occasion could not exceed 60 or 70; but the great penetrating power made full amends for the lowness of the former; notwithstanding the greatest part of it must have been lost for want of a greater opening of the iris, which could not take in the whole pencil of rays, for this could not be less than 7 or 8 tenths of an inch.

Among other instances of the superior effects of penetration into space, I should mention the discovery of an additional 6th satellite of Saturn, on the 28th of August, 1789; and of a 7th, on the 11th of September, in the same year; which were first pointed out by this instrument. It is true that both satellites are within the reach of the 20-feet telescope; but it should be remembered, that when an object is once discovered by a superior power, an inferior one will suffice to see it afterwards. I need not add, that neither the 7 nor 10-feet telescopes will reach them; their powers, 20 and 29, are not sufficient to penetrate to such distant objects, when the brightness of them is not more than that of these satellites. It is also evident, that the failure in these latter instruments, arises not from want of magnifying power; as either of them has much more than sufficient for the purpose.

Nov. 5, 1791. I viewed Saturn with the 20 and 40-feet telescopes.

"20-feet. The 5th satellite of Saturn is very small. The 1st, "2d, 3d, 4th, 5th, and the new 6th satellite, are in their cal-"culated places."

"40-feet. I see the new 6th satellite much better with this "instrument than with the 20-feet. The 5th is also much larger "here than in the 20-feet; in which it was nearly the same size "as a small fixed star, but here it is considerably larger than "that star."

Here the superior penetrating power of the 40-feet telescope shewed itself on the 6th satellite of Saturn, which is a very faint object; as it had also a considerable advantage in magnifying power, the disk of the 5th satellite appeared larger than in the 20-feet. But the small star, which may be said to be beyond the reach of magnifying power, could only profit by the superiority of the other power.

Nov. 21, 1791. 40-feet reflector; power 370.

"The black division upon the ring is as dark as the heavens "about Saturn, and of the same colour."

"The shadow of the body of Saturn is visible upon the ring, "on the following side; its colour is very different from that "of the dark division. The 5th satellite is less than the 3d; it "is even less than the 2d."

20-feet reflector; power 300.

"The 3d satellite seems to be smaller than it was the last night "but one. The 4th satellite seems to be larger than it was the "19th. This telescope shews the satellites not nearly so well "as the 40-feet."

Here, the magnifying power being nearly alike, the superiority of the 40-feet telescope must be ascribed to its penetrating power.

The different nature of the two powers above mentioned being thus evidently established, I must now remark, that, in some respects, they even interfere with each other; a few instances of which I shall give.

August 24, 1783. I viewed the nebula north preceding FLAMSTEED'S 1 Trianguli, discovered by Mr. MESSIER, in 1764.

"7-feet reflector; power 57. There is a suspicion that the "nebula consists of exceedingly small stars. With this low

" power it has a nebulous appearance; and it vanishes when I
" put on the higher magnifying powers of 278 and 460."

Oct. 28, 1794. I viewed the same nebula with a 7-feet reflector.

" It is large, but very faint. With 120, it seems to be com-
" posed of stars, and I think I see several of them; but it will
" bear no magnifying power."

In this experiment, magnifying power was evidently injurious to penetrating power. I do not account for this upon the principle that by magnifying we make an object less bright; for, when opticians have also demonstrated that brightness is diminished by magnifying, it must again be understood as relating only to the *intrinsic* brightness of the magnified picture; its absolute brightness, which is the only one that concerns us at present, must always remain the same.* The real explanation of the fact, I take to be, that while the light collected is employed in magnifying the object, it cannot be exerted in giving penetrating power.

* This may be proved thus. The mean intrinsic brightness, or rather illumination, of a point of the picture on the retina, will be *all the light that falls on the picture, divided by the number of its points*; or $C = \frac{l}{N}$. Now, since with a greater magnifying power m, the number of points N increases as the squares of the power, the expression for the intrinsic brightness $\frac{l}{N}$, will decrease in the same ratio; and it will consequently be in general $N \propto m^2$, and $\frac{l}{N}$ or $C \propto \frac{1}{m^2}$; that is, by compounding $CN \propto \frac{m^2}{m^2} = l = 1$; or absolute brightness a given quantity. M. BOUGUER has carefully distinguished intrinsic and absolute brightness, when he speaks of the quantity of light reflected from a wall, at different distances. *Traité d'Optique*, page 39, and 40.

June 18, 1799. I viewed the planet Venus with a 10-feet reflector.

"Its light is so vivid that it does not require, nor will it bear, "a penetrating power of 29, neither with a low nor with a "high magnifying power."

This is not owing to the least imperfection in the mirror, which is truly parabolical, and shews, with all its aperture open, and a magnifying power of 600, the double star γ Leonis in the greatest perfection.

"It shewed Venus, perfectly well defined, with a penetrating "power as low as 14, and a magnifying power of 400, or 600."

Here, penetrating power was injurious to magnifying power; and that it necessarily must be so, when carried to a high pitch, is evident; for, by enlarging the aperture of the telescope, we increase the evil that attends magnifying, which is, that we cannot magnify the object without magnifying the medium. Now, since the air is very seldom of so homogeneous a disposition as to admit to be magnified highly, it follows that we must meet with impurities and obstructions, in proportion to its quantity. But the contents of the columns of air through which we look at the heavens by telescopes, being of equal lengths, must be as their bases, that is, as the squares of the apertures of the telescopes; and this is in a much higher ratio than that of the increase of the power of penetrating into space. From my long experience in these matters, I am led to apprehend, that the highest power of magnifying may possibly not exceed the reach of a 20 or 25-feet telescope; or may even lie in a less compass than either. However, in beautiful nights, when the outside of our telescopes is dropping with moisture discharged from the atmosphere, there are now and then favourable hours, in which it is hardly possible to put a limit to magnifying power. But such valuable opportunities are extremely scarce; and, with large instruments, it will always be lost labour to observe at other times.

As I have hinted at the natural limits of magnifying power, I shall venture also to extend my surmises to those of penetrating power. There seems to be room for a considerable increase in this branch of the telescope; and, as the penetrating power of my 40-feet reflector already goes to 191,69, there can hardly be any doubt but that it might be carried to 500, and probably not much farther. The natural limit seems to be an equation between the faintest star that can be made visible, by any means, and the united brilliancy of star-light. For, as the light of the heavens, in clear nights, is already very considerable in my large telescope, it must in the end be so increased, by enlarging the penetrating power, as to become a balance to the light of all objects that are so remote as not to exceed in brightness the general light of the heavens. Now, if P be put for penetrating power, we have $\sqrt{\frac{P^2 a^2}{x}} = A =$ 10 feet 5,2 inches for an aperture of a reflector, on my construction, that would have such a power of 500.

But, to return to our subject; from what has been said before, we may conclude, that objects are viewed in their greatest perfection, when, in penetrating space, the magnifying power is so low as only to be sufficient to shew the object well; and when, in magnifying objects, by way of examining them minutely, the space-penetrating power is no higher that what will suffice for the purpose; for, in the use of either power, the injudicious overcharge of the other, will prove hurtful to perfect vision.

It is remarkable that, from very different principles, I have formerly determined the length of the visual ray of my 20-feet telescope upon the stars of the milky way, so as to agree nearly with the calculations that have been given.* The extent of what I then figuratively called my sounding line, and what now appears to answer to the power of penetrating into space, was shewn to be not less than 415, 461, and 497 times the distance of Sirius from the sun. We now have calculated that my telescope, in the NEWTONIAN form, at the time when the paper on the Construction of the Heavens was written, possessed a power of penetration, which exceeded that of natural vision 61,18 times; and, as we have also shewn, that stars at 8, 9, or at most 10 times the distance of Sirius, must become invisible to the eye, we may safely conclude, that no single star, above 489, 551, or at most 612 times as far as Sirius, can any longer be seen in this telescope. Now, the greatest length of the former visual ray, 497, agrees nearly with the lowest of these present numbers, 489; and the higher ones are all in favour of the former computation; for that ray, though taken from what was perhaps not far from its greatest extent, might possibly have reached to some distance beyond the apparent bounds of the milky way: but, if there had been any considerable difference in these determinations, we should remember that some of the data by which I have now calculated are only assumed. For instance, if the opening of the iris, when we look at a star of the 7th magnitude, should be only one-tenth of an inch and a half, instead of two, then a, in our formula, will be $= 1,5$; which, when resolved,

* Phil. Trans. Vol. LXXV. page 247, 248.

will give a penetrating power of 81,58; and therefore, on this supposition, our telescope would easily have shewn stars 571 times as far from us as Sirius; and only those at 653, 734, or 816 times the same distance, would have been beyond its reach. My reason for fixing upon two-tenths, rather than a lower quantity, was, that I might not run a risk of over-rating the powers of my instruments. I have it however in contemplation, to determine this quantity experimentally, and perceive already, that the difficulties which attend this subject may be overcome.

It now only remains to shew, how far the penetrating power, 192, of my large reflector, will really reach into space. Then, since this number has been calculated to be in proportion to the standard of natural vision, it follows, that if we admit a star of the 7th magnitude to be visible to the unassisted eye, this telescope will shew stars of the one thousand three hundred and forty-second magnitude.

But, as we did not stop at the single stars above mentioned, when the penetration of the natural eye was to be ascertained, so we must now also call the united lustre of sidereal systems to our aid in stretching forwards into space. Suppose therefore, a cluster of 5000 stars to be at one of those immense distances to which only a 40-feet reflector can reach, and our formula will give us the means of calculating what that may be. For, putting S for the number of stars in the cluster, and D for its distance, we have $\frac{\sqrt{x A^2 S}}{a} = D$; * which, on computation,

* $D = 11765475948678678679$ miles.

comes out to be above $11\frac{3}{4}$ millions of millions of millions of miles! A number which exceeds the distance of the nearest fixed star, at least three hundred thousand times.

From the above considerations it follows, that the range for observing, with a telescope such as my 40-feet reflector, is indeed very extensive. We have the inside of a sphere to examine, the radius of which is the immense distance just now assigned to be within the reach of the penetration of our instruments, and of which all the celestial objects visible to the eye, put together, form as it were but the kernel, while all the immensity of its thick shell is reserved for the telescope.

It follows, in the next place, that much time must be required for going through so extensive a range. The method of examining the heavens, by sweeping over space, instead of looking merely at places that are known to contain objects, is the only one that can be useful for discoveries.

In order therefore to calculate how long a time it must take to sweep the heavens, as far as they are within the reach of my 40-feet telescope, charged with a magnifying power of 1000, I have had recourse to my journals, to find how many favourable hours we may annually hope for in this climate. It is to be noticed, that the nights must be very clear; the moon absent; no twilight; no haziness; no violent wind; and no sudden change of temperature; then also, short intervals for filling up broken sweeps will occasion delays; and, under all these circumstances, it appears that a year which will afford 90, or at most 100 hours, is to be called very productive.

In the equator, with my 20-feet telescope, I have swept over zones of two degrees, with a power of 157; but, an allowance of 10 minutes in polar distance must be made, for lapping the sweeps over one another where they join.

As the breadth of the zones may be increased towards the poles, the northern hemisphere may be swept in about 40 zones: to these we must add 19 southern zones; then, 59 zones, which, on account of the sweeps lapping over one another about 5′ of time in right ascension, we must reckon of 25 hours each, will give 1475 hours. And, allowing 100 hours per year, we find that, with the 20-feet telescope, the heavens may be swept in about 14 years and $\frac{3}{4}$.

Now, the time of sweeping with different magnifying powers will be as the squares of the powers; and, putting p and t for the power and time in the 20-feet telescope, and $P = 1000$ for the power in the 40, we shall have $p^2 : t :: P^2 : \frac{tP^2}{p^2} = 59840$. Then, making the same allowance of 100 hours per year, it appears that it will require not less than 598 years, to look with the 40-feet reflector, charged with the abovementioned power, only one single moment into each part of space; and, even then, so much of the southern hemisphere will remain unexplored, as will take up 213 years more to examine.

Slough, near Windsor,
June 20, 1799.

V. *A second Appendix to the improved Solution of a Problem in physical Astronomy, inserted in the Philosophical Transactions for the Year* 1798, *containing some further Remarks, and improved* Formulæ *for computing the Coefficients* A *and* B; *by which the arithmetical Work is considerably shortened and facilitated. By the Rev.* John Hellins, *B. D. F. R. S. and Vicar of* Potter's Pury, *in* Northamptonshire.

Read December 12, 1799.

1. It was shewn, in Art. 9. of the first Appendix, that the common logarithm of the fraction $\frac{1+\sqrt{(1-cc)}}{c}$, when c is expressed in numbers, might be taken out from Taylor's excellent tables, and converted into an hyperbolic logarithm by means of table XXXVII. of Dodson's Calculator; which method of obtaining the H. L. $\frac{1+\sqrt{(1-cc)}}{c}$ is undoubtedly easier and shorter than the more obvious one of first computing the numerical value of that fraction, and then taking out the hyperbolic logarithm corresponding to it from a table. But yet, that method of obtaining the value of α, easy as it is, requires, first, a search in the table for the angle of which c is the sine, and generally a proportion for the fractional parts of a second: then, a division of the degrees, minutes, and seconds contained in that angle, by 2; and, thirdly, another search for the logarithmic tangent of half the angle, and another proportion to find the fractional parts of a second. I was therefore desirous